INTERNATIONAL MATHEMATICAL OLYMPIADS

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Hojoo Lee, Version 1.0

Contents

1. Problems

021 Let n be a positive integer. Let T be the set of points (x, y) in the plane where x and y are non-negative integers and $x + y < n$. Each point of T is colored red or blue. If a point (x, y) is red, then so are all points (x', y') of T with both $x' \leq x$ and $y' \leq y$. Define an X-set to be a set of n blue points having distinct x-coordinates, and a Y-set to be a set of n blue points having distinct y-coordinates. Prove that the number of X-sets is equal to the number of Y-sets.

022 Let BC be a diameter of circle ω with center O. Let A be a point of circle ω such that $0^\circ < \angle AOB < 120^\circ$. Let D be the midpoint of arc AB not containing C. Line ℓ passes through O and is parallel to line AD. Line ℓ intersects line AC at J. The perpendicular bisector of segment OA intersects circle ω at E and F. Prove that J is the incenter of triangle CEF.

023 Find all pairs of integers $m, n \geq 3$ such that there exist infinitely many positive integers a for which

$$\frac{a^m + a - 1}{a^n + a^2 - 1}$$

is an integer.

024 Let n be an integer greater than 1. The positive divisors of n are $d_1, d_2, \ldots, d_k$ where $1 = d_1 < d_2 < \cdots < d_k = n$. Define $D = d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k$.
(a) Prove that $D < n^2$.
(b) Determine all n for which D is a divisor of n^2.

025 Find all functions f from the set $\mathbb{R}$ of real numbers to itself such that

$$(f(x) + f(z))(f(y) + f(t)) = f(xy - zt) + f(xt + yz)$$

for all x, y, z, t in $\mathbb{R}$.

026 Let $\Gamma_1, \Gamma_2, \ldots, \Gamma_n$ be circles of radius 1 in the plane, where $n \geq 3$. Denote their centers by $O_1, O_2, \ldots, O_n$ respectively. Suppose that no line meets more than two of the circles. Prove that

$$\sum_{1 \leq i < j \leq n} \frac{1}{O_i O_j} \leq \frac{(n-1)\pi}{4}.$$

011 Let ABC be an acute-angled triangle with O as its circumcenter. Let P on line BC be the foot of the altitude from A. Assume that $\angle BCA \geq \angle ABC + 30^\circ$. Prove that $\angle CAB + \angle COP < 90^\circ$.

012 Prove that

$$\frac{a}{\sqrt{a^2 + 8bc}} + \frac{b}{\sqrt{b^2 + 8ca}} + \frac{c}{\sqrt{c^2 + 8ab}} \geq 1$$

for all positive real numbers a, b, and c.

013 Twenty-one girls and twenty-one boys took part in a mathematical competition. It turned out that

(a) each contestant solved at most six problems, and

(b) for each pair of a girl and a boy, there was at least one problem that was solved by both the girl and the boy.

Prove that there is a problem that was solved by at least three girls and at least three boys.

014 Let n be an odd integer greater than 1 and let $c_1, c_2, \ldots, c_n$ be integers. For each permutation $a = (a_1, a_2, \ldots, a_n)$ of $\{1, 2, \ldots, n\}$, define $S(a) = \sum_{i=1}^{n} c_i a_i$. Prove that there exist permutations b and c, $b \neq c$, such that $n!$ divides $S(b) - S(c)$.

015 In a triangle ABC, let segment AP bisect $\angle BAC$, with P on side BC, and let segment BQ bisect $\angle ABC$, with Q on side CA. It is known that $\angle BAC = 60^\circ$ and that $AB + BP = AQ + QB$. What are the possible angles of triangle ABC?

016 Let $a > b > c > d$ be positive integers and suppose

$$ac + bd = (b + d + a - c)(b + d - a + c).$$

Prove that $ab + cd$ is not prime.

001 Two circles ω_1 and ω_2 intersect at M and N. Line ℓ is tangent to the circles at A and B, respectively, so that M lies closer to ℓ than N. Line CD, with C on ω_1 and D on ω_2, is parallel to ℓ and passes through M. Let lines AC and BD meet at E; let lines AN and CD meet at P; and let lines BN and CD meet at Q. Prove that $EP = EQ$.

002 Let a, b, c be positive real numbers such that $abc = 1$. Prove that

$$\left(a - 1 + \frac{1}{b}\right)\left(b - 1 + \frac{1}{c}\right)\left(c - 1 + \frac{1}{a}\right) \leq 1.$$

003 Let $n \geq 2$ be a positive integer. Initially, there are n fleas on a horizontal line, not all at the same point. For a positive real number λ, define a *move* as follows:

> choose any two fleas, at points A and B, with A to the left of B; let the flea at A jump to the point C on the line to the right of B with $BC/AB = \lambda$.

Determine all values of λ such that, for any point M on the line and any initial positions of the n fleas, there is a finite sequence of moves that will take all the fleas to positions to the right of M.

004 A magician has one hundred cards numbered 1 to 100. He puts them into three boxes, a red one, a white one and a blue one, so that each box contains at least one card. A member of the audience selects two of the three boxes, chooses one card from each and announces the sum of the numbers of the chosen cards. Given this sum, the magician identifies the box from which no card has been chosen. How many ways are there to put all the cards into the boxes so that this trick always works? (Two ways are considered different if at least one card is put into a different box.)

005 Determine if there exists a positive integer n such that n has exactly 2000 prime divisors and $2^n + 1$ is divisible by n.

006 Let $\overline{AH_1}$, $\overline{BH_2}$, and $\overline{CH_3}$ be the altitudes of an acute triangle ABC. The incircle ω of triangle ABC touches the sides BC, CA and AB at T_1, T_2 and T_3, respectively. Consider the symmetric images of the lines H_1H_2, H_2H_3, and H_3H_1 with respect to the lines T_1T_2, T_2T_3, and T_3T_1. Prove that these images form a triangle whose vertices lie on ω.

991 Determine all finite sets S of at least three points in the plane which satisfy the following condition:

for any two distinct points A and B in S, the perpendicular bisector of the line segment AB is an axis of symmetry for S.

992 Let n be a fixed integer, with $n \geq 2$.

(a) Determine the least constant C such that the inequality

$$\sum_{1 \leq i < j \leq n} x_i x_j (x_i^2 + x_j^2) \leq C \left(\sum_{1 \leq i \leq n} x_i \right)^4$$

holds for all real numbers $x_1, \cdots, x_n \geq 0$.

(b) For this constant C, determine when equality holds.

993 Consider an $n \times n$ square board, where n is a fixed even positive integer. The board is divided into n^2 unit squares. We say that two different squares on the board are adjacent if they have a common side. N unit squares on the board are marked in such a way that every square (marked or unmarked) on the board is adjacent to at least one marked square. Determine the smallest possible value of N.

994 Determine all pairs (n, p) of positive integers such that

p is a prime,
n not exceeded $2p$, and
$(p-1)^n + 1$ is divisible by n^{p-1}.

995 Two circles G_1 and G_2 are contained inside the circle G, and are tangent to G at the distinct points M and N, respectively. G_1 passes through the center of G_2. The line passing through the two points of intersection of G_1 and G_2 meets G at A and B. The lines MA and MB meet G_1 at C and D, respectively. Prove that CD is tangent to G_2.

996 Determine all functions $f : \mathbf{R} \longrightarrow \mathbf{R}$ such that

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

for all real numbers x, y.

981 In the convex quadrilateral $ABCD$, the diagonals AC and BD are perpendicular and the opposite sides AB and DC are not parallel. Suppose that the point P, where the perpendicular bisectors of AB and DC meet, is inside $ABCD$. Prove that $ABCD$ is a cyclic quadrilateral if and only if the triangles ABP and CDP have equal areas.

982 In a competition, there are a contestants and b judges, where $b \geq 3$ is an odd integer. Each judge rates each contestant as either "pass" or "fail". Suppose k is a number such that, for any two judges, their ratings coincide for at most k contestants. Prove that

$$\frac{k}{a} \geq \frac{b-1}{2b}.$$

983 For any positive integer n, let $d(n)$ denote the number of positive divisors of n (including 1 and n itself). Determine all positive integers k such that

$$\frac{d(n^2)}{d(n)} = k$$

for some positive integer n.

984 Determine all pairs (a, b) of positive integers such that $ab^2 + b + 7$ divides $a^2b + a + b$.

985 Let I be the incenter of triangle ABC. Let the incircle of ABC touch the sides BC, CA, and AB at K, L, and M, respectively. The line through B parallel to MK meets the lines LM and LK at R and S, respectively. Prove that angle RIS is acute.

986 Consider all functions f from the set N of all positive integers into itself satisfying $f(t^2f(s)) = s(f(t))^2$ for all s and t in N. Determine the least possible value of $f(1998)$.

971 In the plane the points with integer coordinates are the vertices of unit squares. The squares are colored alternately black and white (as on a chessboard). For any pair of positive integers m and n, consider a right-angled triangle whose vertices have integer coordinates and whose legs, of lengths m and n, lie along edges of the squares. Let S_1 be the total area of the black part of the triangle and S_2 be the total area of the white part. Let

$$f(m, n) = |S_1 - S_2|.$$

(a) Calculate $f(m, n)$ for all positive integers m and n which are either both even or both odd.
(b) Prove that $f(m, n) \leq \frac{1}{2}\max\{m, n\}$ for all m and n.
(c) Show that there is no constant C such that $f(m, n) < C$ for all m and n.

972 The angle at A is the smallest angle of triangle ABC. The points B and C divide the circumcircle of the triangle into two arcs. Let U be an interior point of the arc between B and C which does not contain A. The perpendicular bisectors of AB and AC meet the line AU at V and W, respectively. The lines BV and CW meet at T. Show that

$$AU = TB + TC.$$

973 Let $x_1, x_2, \ldots, x_n$ be real numbers satisfying the conditions

$$|x_1 + x_2 + \cdots + x_n| = 1$$

and

$$|x_i| \leq \frac{n+1}{2} \qquad i = 1, 2, \ldots, n.$$

Show that there exists a permutation $y_1, y_2, \ldots, y_n$ of $x_1, x_2, \ldots, x_n$ such that

$$|y_1 + 2y_2 + \cdots + ny_n| \leq \frac{n+1}{2}.$$

974 An $n \times n$ matrix whose entries come from the set $S = \{1, 2, \ldots, 2n - 1\}$ is called a *silver* matrix if, for each $i = 1, 2, \ldots, n$, the ith row and the ith column together contain all elements of S. Show that

(a) there is no silver matrix for $n = 1997$;
(b) silver matrices exist for infinitely many values of n.

975 Find all pairs (a, b) of integers $a, b \geq 1$ that satisfy the equation

$$a^{b^2} = b^a.$$

976 For each positive integer n , let $f(n)$ denote the number of ways of representing n as a sum of powers of 2 with nonnegative integer exponents. Representations which differ only in the ordering of their summands are considered to be the same. For instance, $f(4) = 4$, because the number 4 can be represented in the following four ways:

$$4; 2+2; 2+1+1; 1+1+1+1.$$

Prove that, for any integer $n \geq 3$,

$$2^{n^2/4} < f(2^n) < 2^{n^2/2}.$$

961 We are given a positive integer r and a rectangular board $ABCD$ with dimensions $AB = 20, BC = 12$. The rectangle is divided into a grid of 20×12 unit squares. The following moves are permitted on the board: one can move from one square to another only if the distance between the centers of the two squares is $\sqrt{r}$. The task is to find a sequence of moves leading from the square with A as a vertex to the square with B as a vertex.

(a) Show that the task cannot be done if r is divisible by 2 or 3.
(b) Prove that the task is possible when $r = 73$.
(c) Can the task be done when $r = 97$?

962 Let P be a point inside triangle ABC such that

$$\angle APB - \angle ACB = \angle APC - \angle ABC.$$

Let D, E be the incenters of triangles APB, APC, respectively. Show that AP, BD, CE meet at a point.

963 Let S denote the set of nonnegative integers. Find all functions f from S to itself such that

$$f(m + f(n)) = f(f(m)) + f(n) \qquad \text{for all } m, n \in S.$$

964 The positive integers a and b are such that the numbers $15a + 16b$ and $16a - 15b$ are both squares of positive integers. What is the least possible value that can be taken on by the smaller of these two squares?

965 Let $ABCDEF$ be a convex hexagon such that AB is parallel to DE, BC is parallel to EF, and CD is parallel to FA. Let R_A, R_C, R_E denote the circumradii of triangles FAB, BCD, DEF, respectively, and let P denote the perimeter of the hexagon. Prove that

$$R_A + R_C + R_E \geq \frac{P}{2}.$$

966 Let p, q, n be three positive integers with $p+q < n$. Let $(x_0, x_1, \ldots, x_n)$ be an $(n+1)$-tuple of integers satisfying the following conditions :

(a) $x_0 = x_n = 0$, and
(b) For each i with $1 \leq i \leq n$, either $x_i - x_{i-1} = p$ or $x_i - x_{i-1} = -q$.

Show that there exist indices $i < j$ with $(i, j) \neq (0, n)$, such that $x_i = x_j$.

951 Let A, B, C, D be four distinct points on a line, in that order. The circles with diameters AC and BD intersect at X and Y. The line XY meets BC at Z. Let P be a point on the line XY other than Z. The line CP intersects the circle with diameter AC at C and M, and the line BP intersects the circle with diameter BD at B and N. Prove that the lines AM, DN, XY are concurrent.

952 Let a, b, c be positive real numbers such that $abc = 1$. Prove that
$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \geq \frac{3}{2}.$$

953 Determine all integers $n > 3$ for which there exist n points $A_1, \dots, A_n$ in the plane, no three collinear, and real numbers $r_1, \dots, r_n$ such that for $1 \leq i < j < k \leq n$, the area of $\triangle A_iA_jA_k$ is $r_i + r_j + r_k$.

954 Find the maximum value of x_0 for which there exists a sequence $x_0, x_1 \dots, x_{1995}$ of positive reals with $x_0 = x_{1995}$, such that for $i = 1, \dots, 1995$,
$$x_{i-1} + \frac{2}{x_{i-1}} = 2x_i + \frac{1}{x_i}.$$

955 Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$ and $DE = EF = FA$, such that $\angle BCD = \angle EFA = \frac{\pi}{3}$. Suppose G and H are points in the interior of the hexagon such that $\angle AGB = \angle DHE = 2\pi/3$. Prove that $AG + GB + GH + DH + HE \geq CF$.

956 Let p be an odd prime number. How many p-element subsets A of $\{1, 2, \dots\ 2p\}$ are there, the sum of whose elements is divisible by p

941 Let m and n be positive integers. Let $a_1, a_2, \dots, a_m$ be distinct elements of $\{1, 2, \dots, n\}$ such that whenever $a_i + a_j \leq n$ for some i, j, $1 \leq i \leq j \leq m$, there exists k, $1 \leq k \leq m$, with $a_i + a_j = a_k$. Prove that
$$\frac{a_1 + a_2 + \cdots + a_m}{m} \geq \frac{n+1}{2}.$$

942 ABC is an isosceles triangle with $AB = AC$. Suppose that

(a) M is the midpoint of BC and O is the point on the line AM such that OB is perpendicular to AB,

(b) Q is an arbitrary point on the segment BC different from B and C,

(c) E lies on the line AB and F lies on the line AC such that E, Q, F are distinct and collinear.

Prove that OQ is perpendicular to EF if and only if $QE = QF$.

943 For any positive integer k, let $f(k)$ be the number of elements in the set $\{k+1, k+2, \dots, 2k\}$ whose base 2 representation has precisely three 1s.

(a) Prove that, for each positive integer m, there exists at least one positive integer k such that $f(k) = m$.
(b) Determine all positive integers m for which there exists exactly one k with $f(k) = m$.

944 Determine all ordered pairs (m, n) of positive integers such that

$$\frac{n^3 + 1}{mn - 1}$$

is an integer.

945 Let S be the set of real numbers strictly greater than -1. Find all functions $f : S \to S$ satisfying the two conditions :

(a) $f(x + f(y) + xf(y)) = y + f(x) + yf(x)$ for all $x, y \in S$;
(b) $\frac{f(x)}{x}$ is strictly increasing on each of the intervals $-1 < x < 0$ and $0 < x$.

946 Show that there exists a set A of positive integers with the following property: For any infinite set S of primes there exist two positive integers $m \in A$ and $n \notin A$ each of which is a product of k distinct elements of S for some $k \geq 2$.

931 Let $f(x) = x^n + 5x^{n-1} + 3$, where $n > 1$ is an integer. Prove that $f(x)$ cannot be expressed as the product of two nonconstant polynomials with integer coefficients.

932 Let D be a point inside acute triangle ABC such that $\angle ADB = \angle ACB + \pi/2$ and $AC \cdot BD = AD \cdot BC$.

(a) Calculate the ratio $(AB \cdot CD)/(AC \cdot BD)$.
(b) Prove that the tangents at C to the circumcircles of $\triangle ACD$ and $\triangle BCD$ are perpendicular.

933 On an infinite chessboard, a game is played as follows. At the start, n^2 pieces are arranged on the chessboard in an n by n block of adjoining squares, one piece in each square. A move in the game is a jump in a horizontal or vertical direction over an adjacent occupied square to an unoccupied square immediately beyond. The piece which has been jumped over is removed. Find those values of n for which the game can end with only one piece remaining on the board.

934 For three points P, Q, R in the plane, we define $m(PQR)$ as the minimum length of the three altitudes of $\triangle PQR$. (If the points are collinear, we set $m(PQR) = 0$.) Prove that for points A, B, C, X in the plane,

$$m(ABC) \leq m(ABX) + m(AXC) + m(XBC).$$

935 Does there exist a function $f : \mathbf{N} \to \mathbf{N}$ such that $f(1) = 2$, $f(f(n)) = f(n) + n$ for all $n \in \mathbf{N}$, and $f(n) < f(n+1)$ for all $n \in \mathbf{N}$?

936 There are n lamps $L_0, \ldots, L_{n-1}$ in a circle ($n > 1$), where we denote $L_{n+k} = L_k$. (A lamp at all times is either on or off.) Perform steps $s_0, s_1, \ldots$ as follows: at step s_i, if L_{i-1} is lit, switch L_i from on to off or vice versa, otherwise do nothing. Initially all lamps are on. Show that

(a) There is a positive integer $M(n)$ such that after $M(n)$ steps all the lamps are on again, (b) If $n = 2^k$, we can take $M(n) = n^2 - 1$,
(c) If $n = 2^k + 1$, we can take $M(n) = n^2 - n + 1$.

921 Find all integers a, b, c with $1 < a < b < c$ such that

$$(a-1)(b-1)(c-1) \quad \text{is a divisor of}\, abc - 1.$$

922 Let $\mathbf{R}$ denote the set of all real numbers. Find all functions $f : \mathbf{R} \to \mathbf{R}$ such that

$$f\left(x^2 + f(y)\right) = y + (f(x))^2 \quad \text{for all}\, x, y \in R.$$

923 Consider nine points in space, no four of which are coplanar. Each pair of points is joined by an edge (that is, a line segment) and each edge is either colored blue or red or left uncolored. Find the smallest value of n such that whenever exactly n edges are colored, the set of colored edges necessarily contains a triangle all of whose edges have the same color.

924 In the plane let C be a circle, L a line tangent to the circle C, and M a point on L. Find the locus of all points P with the following property: there exists two points Q, R on L such that M is the midpoint of QR and C is the inscribed circle of triangle PQR.

925 Let S be a finite set of points in three-dimensional space. Let S_x, S_y, S_z be the sets consisting of the orthogonal projections of the points of S onto the yz-plane, zx-plane, xy-plane, respectively. Prove that

$$|S|^2 \leq |S_x| \cdot |S_y| \cdot |S_z|,$$

where $|A|$ denotes the number of elements in the finite set A. (Note : The orthogonal projection of a point onto a plane is the foot of the perpendicular from that point to the plane.)

926 For each positive integer n, $S(n)$ is defined to be the greatest integer such that, for every positive integer $k \leq S(n)$, n^2 can be written as the sum of k positive squares.

(a) Prove that $S(n) \leq n^2 - 14$ for each $n \geq 4$.
(b) Find an integer n such that $S(n) = n^2 - 14$.
(c) Prove that there are infintely many integers n such that $S(n) = n^2 - 14$

911 Given a triangle ABC, let I be the center of its inscribed circle. The internal bisectors of the angles A, B, C meet the opposite sides in A', B', C' respectively. Prove that

$$\frac{1}{4} < \frac{AI \cdot BI \cdot CI}{AA' \cdot BB' \cdot CC'} \leq \frac{8}{27}.$$

912 Let $n > 6$ be an integer and $a_1, a_2, \ldots, a_k$ be all the natural numbers less than n and relatively prime to n. If

$$a_2 - a_1 = a_3 - a_2 = \cdots = a_k - a_{k-1} > 0,$$

prove that n must be either a prime number or a power of 2.

913 Let $S = \{1, 2, 3, \ldots, 280\}$. Find the smallest integer n such that each n-element subset of S contains five numbers which are pairwise relatively prime.

914 Suppose G is a connected graph with k edges. Prove that it is possible to label the edges $1, 2, \ldots, k$ in such a way that at each vertex which belongs to two or more edges, the greatest common divisor of the integers labeling those edges is equal to 1.

[A *graph* consists of a set of points, called *vertices*, together with a set of *edges* joining certain pairs of distinct vertices. Each pair of vertices u, v belongs to at most one edge. The graph G is *connected* if for each pair of distinct vertices x, y there is some sequence of vertices $x = v_0, v_1, v_2, \ldots, v_m = y$ such that each pair v_i, v_{i+1} $(0 \le i < m)$ is joined by an edge of G.]

915 Let ABC be a triangle and P an interior point of ABC. Show that at least one of the angles $\angle PAB$, $\angle PBC$, $\angle PCA$ is less than or equal to $30°$.

916 An infinite sequence $x_0, x_1, x_2, \ldots$ of real numbers is said to be *bounded* if there is a constant C such that $|x_i| \le C$ for every $i \ge 0$. Given any real number $a > 1$, construct a bounded infinite sequence $x_0, x_1, x_2, \ldots$ such that

$$|x_i - x_j||i - j|^a \ge 1$$

for every pair of distinct nonnegative integers i, j.

901 Chords AB and CD of a circle intersect at a point E inside the circle. Let M be an interior point of the segment EB. The tangent line at E to the circle through D, E, and M intersects the lines BC and AC at F and G, respectively. If

$$\frac{AM}{AB} = t,$$

find

$$\frac{EG}{EF}$$

in terms of t.

902 Let $n \ge 3$ and consider a set E of $2n - 1$ distinct points on a circle. Suppose that exactly k of these points are to be colored black. Such a coloring is "good" if there is at least one pair of black points such that the interior of one of the arcs between them contains exactly n points from E. Find the smallest value of k so that every such coloring of k points of E is good.

903 Determine all integers $n > 1$ such that

$$\frac{2^n + 1}{n^2}$$

is an integer.

904 Let $\mathbb{Q}^+$ be the set of positive rational numbers. Construct a function $f : \mathbb{Q}^+ \to \mathbb{Q}^+$ such that

$$f(xf(y)) = \frac{f(x)}{y}$$

for all x, y in $\mathbb{Q}^+$.

905 Given an initial integer $n_0 > 1$, two players, $\mathcal{A}$ and $\mathcal{B}$, choose integers n_1, n_2, n_3, $\ldots$ alternately according to the following rules :

(a) Knowing n_{2k}, $\mathcal{A}$ chooses any integer n_{2k+1} such that

$$n_{2k} \leq n_{2k+1} \leq n_{2k}^2.$$

(b) Knowing n_{2k+1}, $\mathcal{B}$ chooses any integer n_{2k+2} such that

$$\frac{n_{2k+1}}{n_{2k+2}}$$

is a prime raised to a positive integer power.

Player $\mathcal{A}$ wins the game by choosing the number 1990; player $\mathcal{B}$ wins by choosing the number 1. For which n_0 does :

(a) $\mathcal{A}$ have a winning strategy?
(b) $\mathcal{B}$ have a winning strategy?
(c) Neither player have a winning strategy?

906 Prove that there exists a convex 1990-gon with the following two properties :

(a) All angles are equal.
(b) The lengths of the 1990 sides are the numbers 1^2, 2^2, 3^2, $\ldots$, 1990^2 in some order.

2. Answers and Hints

021
022
023 $(m,n)=(5,3)$
024
025 $f(x)=0,\frac{1}{2},x^2$
026
011
012
013
014
015
016
001
002
003 $\lambda \geq \frac{1}{n-1}$
004 12
005 Yes
006
991
992 $\frac{1}{8}$
993
994 $(2,2),(3,3),(1,p)$ (p is prime.)
995
996 $f(x)=1-\frac{x^2}{2}$
981
982
983 $n \equiv 1\ (mod\ 2)$
984 $(11,1)$, $(49,1)$
985
986 120
971 $f(m,n)=\frac{1}{2}\ (m \equiv n \equiv 1\ (mod\ 2)),\ f(m,n)=0$ (otherwise)
972
973
974
975 $(1,2)$, $(27,3)$, $(16,2)$
976
961
962
963
964 481^2
965
966

951
952
953 $n = 4$
954 2^{997}
955
956 $\frac{1}{p}\left(\binom{2p}{p} - 2\right)$
941
942
943
944 $(2,2),(1,2),(1,3),(2,5),(3,5),(2,1),(3,1),(5,2),(5,3)$
945 $f(x) = \frac{-x}{1+x}$
946
931
932 $\sqrt{2}$
933 $3 \nmid n$
934
935 Yes
936
921 $(2,4,8),(3,5,15)$
922
923
924
925
926 (b) $n = 13$
911
912
913 217
914
915
916
901 $\frac{t}{1-t}$
902 $k = n - 1$ $(3 \mid 2n-1)$, $k = n$ (otherwise)
903 $n = 3$
904
905
906

3. References

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